Eighth Week.
Trigonometry - Inductance - Capacity.

QUESTION #1. What is the nature of trigonometry?

ANSWER #1. Trigonometry is a branch of mathematics using numbers though not chiefly relating to numbers, not exclusively devoted to equations though using them, not concerned with geometry altho drawing freely upon geometric facts. Trigonometry is mostly concerned with relation of certain lines in a triangle which form the basis of mensuration used in surveying, engineering, mechanics, geodesy, and astronomy.

QUESTION #2. What is meant by an angle? What unit is used for measuring angles?

ANSWER #2. An angle is the result of an intersection of two lines that is, when two lines meet they form an angle. The unit used to measure angles is the degree (°).

QUESTION #3. How many degrees in a right angle? Straight angle?

ANSWER #3. There are 90° in a right angle. A right angle is one whose sides are distinctly at right angles to each other. A straight angle is equal to two right angles so that it may be represented by a straight line. A straight angle has 180°.

QUESTION #4. Name and briefly describe the instrument used for the measurement of angles.

ANSWER #4. For ordinary purposes angles can be measured with a protractor to a degree of accuracy of about 30'. A protractor is a semicircle which is accurately calibrated in degrees. There being 360° in a full circle, there are 180° degrees represented by a protractor. The numbering starts from both the left and the right side, so that any angle to the left or to the right of 90° may be measured.

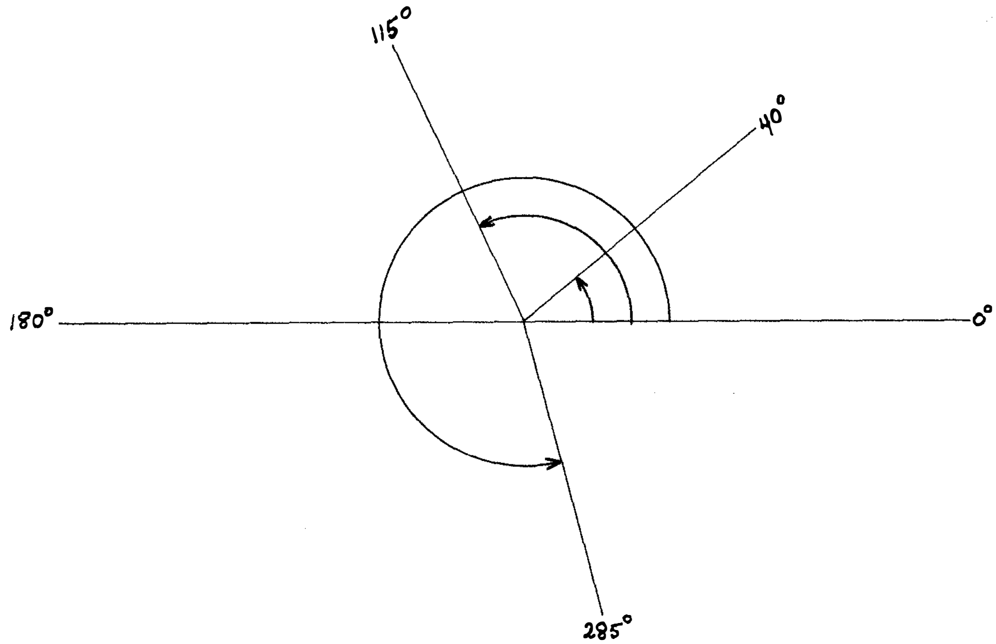
QUESTION #5. Explain how angles are laid off.

ANSWER #5. To lay off any angle using a protractor, first draw a line on the left upper side of the paper, place a dot about the middle of the line, place the protractor so that the intersection of the vertical and horizontal lines, in the center bottom of the protractor, is exactly on the dot, and the zero mark is on the line at either end of the protractor. With a hard pencil, then point off the required angle from the scale on the protractor, remove the protractor and draw a line between the dot on the bottom line and the dot made at the requ-ired angle.

QUESTION #6. With the use of a protractor, lay off the following angles: 40°, 115° and 285°.

ANSWER #6. See next sheet.

ANSWER #6. Continued.



QUESTION #7. What is meant by right triangle?

ANSWER #7. A right triangle has one angle of 90°, the other two angles being acute and complementary to each other, the total sum of all the angles being 180°.

QUESTION #8. What is the sum of the angles in a right triangle? What is the sum of the acute angles in a right triangle

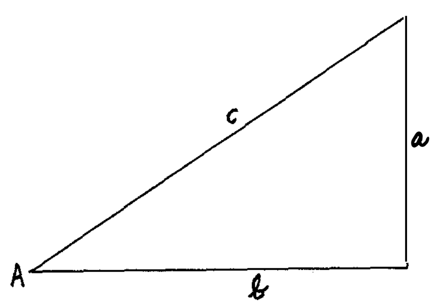
ANSWER #8. The sum of all the angles in a right triangle is 180° The sum of the acute angles in a right triangle is 90° so to find the other acute angle if one is know, sub-tract the known angle from 90°. These acute angles in a right triangle are said to be complements of each other.

QUESTION #9. What is meant by complementary angles?

ANSWER #9. Two angles whose sum is a right angle (or 90°) are cal led complementary angles. Each of the angles is called the complement of the other.

QUESTION #10. Draw a right triangle and designate the sides.

ANSWER #10.



a = altitude
b = base
c = hypotenuse
A = vertex of angle.

QUESTION #11. How are the sides of a right triangle designated when referring to one of the angles?

ANSWER #11. When referring to one of the angles of a right triangle, the sides are called, the side opposite, the side adjacent, and the hypotenuse. When referring to angle A in diagram Question #10, a is the side opposite, b is the side adjacent, and c is the hypotenuse.

QUESTION #12. How is the length of the hypotenuse found when the length of the base and altitude are known?

ANSWER #12. Formula: (basic) $c^2 = a^2 \text{ plus } b^2$

therefore: $c = \sqrt{a^2 \text{ plus } b^2}$

where:
c = hypotenuse.
a = altitude.
b = base.

QUESTION #13. Solve for length of hypotenuse by equation and verify by forming a right triangle to scale: Length of base equals 7.55 centimeters, length of perpendicular equals 4.85 centimeters.

ANSWER #13. Formula: $c = \sqrt{a^2 \text{ plus } b^2}$

$c = \sqrt{4.85^2 \text{ plus } 7.55^2}$

$c = \sqrt{80.5250} = 8.97$

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   8. 9 7
  √80.5250
   64
   16 52
   15 21
    1 3150
    1 2502
       648
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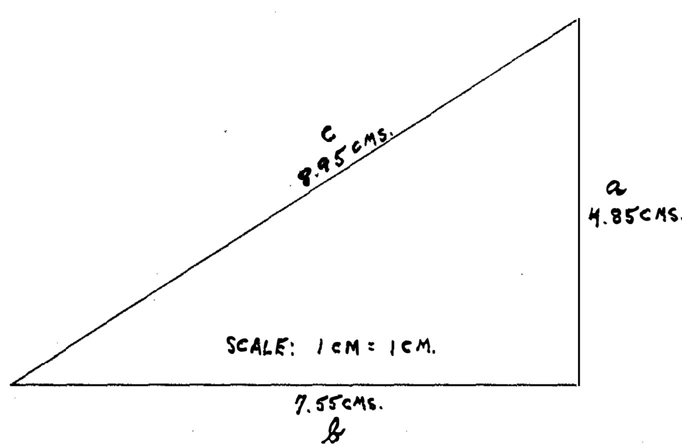
Therefore: Hypotenuse is 8.97 centimeters long. Ans.

Verification by right triangle:



QUESTION #14. How is the length of the base or perpendicular found when the length of the hypotenuse and one side is known?

ANSWER #14. Basic Formula: $c^2 = a^2$ plus b^2

Transposing: $b^2 = c^2$ minus a^2

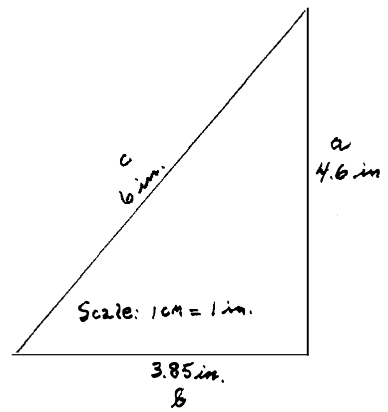
Therefore: $b = \sqrt{c^2 \text{ minus } a^2}$

Similarly: $a^2 = c^2$ minus b^2

Therefore: $a = \sqrt{c^2 \text{ minus } b^2}$

QUESTION #15. Solve for length of the base by forming a right triangle and verify by equation: hypotenuse equals 6 inches, perpendicular equals 4.6 inches.

ANSWER #15.



Verification by equation:

Formula: $b = \sqrt{c^2 \text{ minus } a^2}$

$b = \sqrt{6^2 \text{ minus } 4.6^2}$

$b = \sqrt{36 \text{ minus } 21.16}$

$b = \sqrt{14.84'00}$ = 3. 8 5

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   3. 8  5
 √14.84'00
   9
   5 84
   5 44
     40 00
     38 25
      1 75
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Therefore: base is <u>3.85 inches long. Ans.</u>

QUESTION #16. Show by right triangle and by equation the value of the following functions of an angle: sine, cosine, tangent, and cotangent.

ANSWER #16. See next sheet.

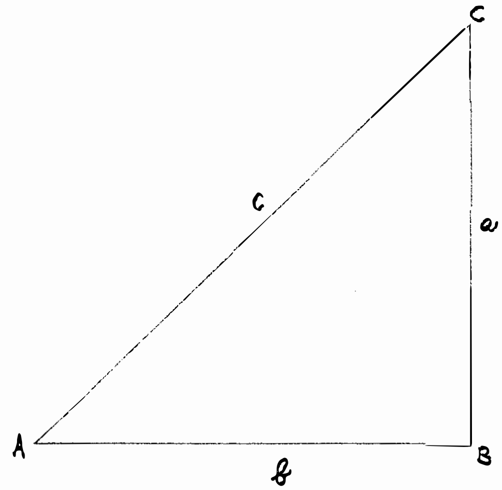
ANSWER #16. Continued.

$$\text{Sin A} = \frac{\text{Side opposite}}{\text{Hypotenuse}} = \frac{a}{c}$$

$$\text{Cos A} = \frac{\text{Side Adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$$

$$\text{Tan A} = \frac{\text{Side opposite}}{\text{Side adjacent}} = \frac{a}{b}$$

$$\text{Cotan A} = \frac{\text{Side adjacent}}{\text{Side opposite}} = \frac{b}{a}$$

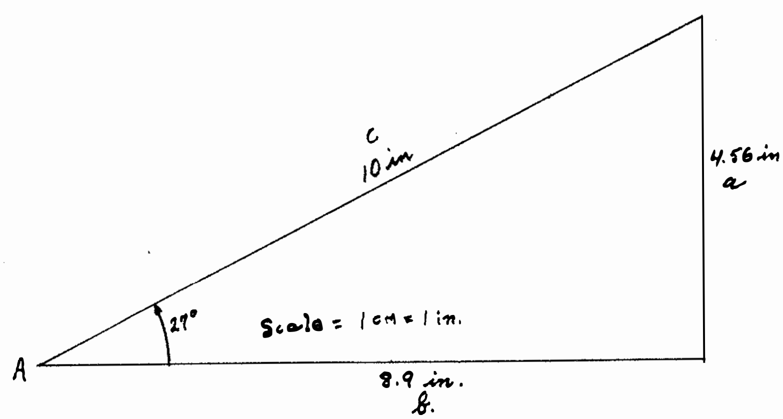


QUESTION #17. Solve by equation and by right triangle: If a right triangle has a hypotenuse 10 inches long and one angle is 27°, what are the lengths of the other two sides?

ANSWER #17.

Sin A 27° = .4540	Cos A 27° = .8910
$\frac{a}{c}$ = .4540	$\frac{b}{c}$ = .8910
$\frac{a}{10}$ = .4540	$\frac{b}{c}$ = .8910
a = 10 x .4540	b = 10 x .8910
a = <u>4.54</u> Ans.	b = <u>8.91</u> Ans.

Verification by plotting right triangle:



QUESTION #18. If the sine of an angle is .487 and the opposite side is 12 inches, what is the length of the hypotenuse?

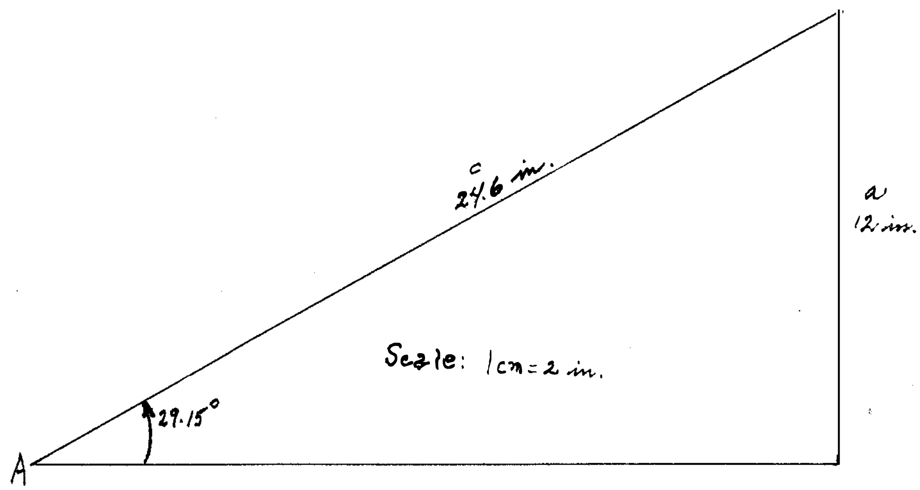
ANSWER #18. Sin A = .487   a = 12 inches.

$$\frac{a}{c} = \text{Sin A}$$

$$c = \frac{a}{\text{Sin A}}$$

$$c = \frac{12}{.487}$$

c = <u>24.64 inches. Ans.</u>



QUESTION #19. What is the cosine of 45°? Tangent of 67°? Sine of 15°

ANSWER #19. The cosine of 45° is 0.7071.
The tangent of 67° is 2.3559.
The sine of 15° is 0.2588.

QUESTION #20. If the tangent of an angle is 2.75 and the adjacent side is 4 centimeters, what is the hypotenuse of the right triangle?

ANSWER #20. We know the tangent of the angle, but in order to find the length of the hypotenuse, we must know one of the functions which includes the hypotenuse, so, we look in the table to find the angle which has a tangent of 2.75. We find it to be 70° and 1'. Under the same angle we find the cosine, .3417.

$$\frac{b}{c} = \cos A \qquad c = \frac{4}{.3417}$$

$$c = \frac{b}{\cos A} \qquad c = 11.706 = \underline{11.71 \text{ cms.Ans.}}$$

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Notes on Trigonometry.

Each function of an acute angle is equal to the co-named function of the complementary angle. Co-sine means complements sine, and similarly for the other co-functions. It is therefore seen that sin 75° equals cos (90° - 75°) equals cos 15°, etc. Therefore any function of an angle between 45° and 90° may be found by taking the co-named function of the complementary angle, which is between 0° and 45°. Hence we need never have a direct table of functions beyond 45°.

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The values of the functions have been computed and tables constructed giving these values. This table gives the values of the functions to four decimal places for every degree from 0° to 90°. All such values are only approximate, the values of the functions, being, in general incommensurable with unity and not being expressible by means of common fractions or by means of decimal fractions with a finite number of decimal places. The table is so arranged that the column of sines from 0° to 45° is the same as the column of cosines from 45° to 90° Therefore, in finding the functions of angles from 0° to 45° read from the top down; in finding the functions of angles from 45° to 90° read from the bottom up.

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The sine is the reciprocal of the cosecant, the cosine is the reciprocal of the secant, and the tangent is the reciprocal of the cotangent.

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Relations of functions: $\sin^2 A$ plus $\cos^2 A$ equals 1; tan A equals sin A divided by cos A; 1 plus $\tan^2 A$ equals $\sec^2 A$; 1 plus $\cot^2 A$ equals $\csc^2 A$. From these formulae the following relations can be easily deduced:

	tan x	
sin x		sec x
cos x		csc x
	cot x	

In the above diagram, any function is equal to the product of the two adjacent functions, or to the quotient of either adjacent function divided by the one beyond it.

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Since the tables give only the sine, cosine, tangent, and cotangent, the proper functions to use, given one side and one angle, is as follows:

To Find.	Given.	Use.	Formula.	Transposition.
a	A & b	Tan A	$\frac{a}{b} = \text{Tan } A$	$a = b \times \text{Tan } A$
c	A & b	Cos A	$\frac{b}{c} = \text{Cos } A$	$c = \frac{b}{\text{Tan } A}$
a	A & c	Sin A	$\frac{a}{c} = \text{Sin } A$	$a = c \times \text{Sin } A$
b	A & c	Cos A	$\frac{b}{c} = \text{Cos } A$	$b = c \times \text{Cos } A$
b	A & a	Cot A	$\frac{b}{a} = \text{Cot } A$	$b = a \times \text{Cot } A$
c	A & a	Sin A	$\frac{a}{c} = \text{Sin } A$	$c = \frac{a}{\text{Sin } A}$

A plane angle is the opening between two straight lines drawn from the same point. The sides are the straight lines and the vertex is their point of meeting.

A perpendicular to a straight line is the straight line that makes a right angle with it. The foot of the perpendicular is the point at which it meets the other straight line.

A straight angle is a straight line and contains 180°.
A right angle is half a straight angle, or 90°
An acute angle is less than 90° in magnitude.
An obtuse angle is greater than 90° in magnitude.
Complementary angles are such that their sum equals 90°
Supplementary angles are such that their sum equals 180°

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A triangle is a portion of a plane included within three straight lines. The straight lines are the sides; their sum is the perimeter; the angles are the angles of the triangle; and their vertices are the vertices of the triangle.

A scalene triangle has no two sides equal.
An isosceles triangle has two sides equal.
An equilateral triangle has all three sides equal.
A right triangle has one angle a right angle.
An obtuse triangle has one obtuse angle.
An acute triangle has all three angles acute angles.
An equiangular triangle has all three angles equal.
The hypotenuse is the side opposite the right angle of a right triangle. The other two sides are the legs.
The base of the triangle is the side on which the triangle stands.
The vertical angle is the angle opposite the base.
The altitude is the perpendicular from the vertex to the base, or base produced.
The sum of the angles of a triangle is equal to 180°

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Useful Formulae:

Notation:

a = one leg of triangle.
b = base of triangle or parallelogram.
c = other leg of triangle.
d = diameter of circle or sphere.
h = perpendicular of triangle of parallelogram.
l = altitude of cylinder.
r = radius of circle or sphere.
S = area.
V = volume.

Linear Values.

Right triangle	a^2 plus b^2	$= c^2$
Circumference of circle		= 2pir or pi x d

Areas.

Rectangle	S	= bh
Square	S	$= b^2$
Parallelogram	S	= bh
Triangle	S	$= \frac{1}{2}bh$
Triangle	S	$= \frac{abc}{4r}$ (r of circumscribed circle)

Area. (Continued).

Equilateral triangle	$S = \frac{a^2}{4}\sqrt{3}$
Circle	$S = pi\ r^2$ or $\frac{pi \times d^2}{4}$
Cylinder (lateral area)	$S = 2pirl$
Cylinder (total area)	$S = 2pir\ (l \text{ plus } r)$
Sphere	$S = 4pir^2$ or pid^2
Cube	$S = 6b^2$

Volumes.

Cube	$V = b^3$
Cylinder	$V = pir^2l$
Sphere	$V = \frac{4pir^3}{3}$ or $\frac{pid^3}{6}$
Cone	$V = \frac{pir^2l}{3}$

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Sexagesimal and Decimal Fractions. The ancients, not having developed the idea of the decimal fraction and not having any convenient notation for even the common fraction, used a system based upon sixtieths. Thus the had units, sixtieths, thirty-six hundredths, and so on, and they used this system in all kinds of theoretical work requiring extensive fractions.

For example, instead of $1\frac{7}{15}$ they would use 1 28' meaning $1\frac{28}{60}$ and instead of 1.51 they would use 1 30'36'' meaning $1\frac{30}{60}$ plus $\frac{36}{3600}$. The symbols for degrees, minutes and seconds are modern. We today apply these sexagesimal fractions only to the measure of time, angles, and arcs.

Since about the year 1600 we have had decimal fractions with which to work, and these have gradually replaced sexagesimal fractions in most cases. At present there is a strong tendency towards using decimal instead of sexagesimal fractions in angle measure. On this account it is necessary to be familiar with tables which give the functions of angles not only to degrees and minutes, but also to degrees and hundredths, with provision for finding the functions to seconds and to thousandths of a degree. Hence the tables which will be considered and the problems which will be proposed will involve both sexagesimal and decimal fractions, but with particular attention to the former because they are the ones which are still commonly used. The rise of the metric system in the nineteenth century gave an impetus to the movement to abandon the sexagesimal system. At the time the metric system was established in France, trigonometric tables were prepared on the decimal plan. It is only within recent years, however that tables of this kind have begun to come into use.

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It has already been seen that the trigonometric functions are incommensurable with unity. Hence they contain decimal fractions of an infinite number of places. Even if we express these fractions only to four or five decimal places, the labor of multiplying and dividing them is considerable. In order to simplify this work, logarithms are used. By their use the operation of multiplication is reduced to that of addition; that of division is reduced to subtraction; raising to any power is reduced to one multiplication; and the extracting of any root is reduced to a single division. Tables are prepared giving the logarithmic functions of angles.

TABLE OF TRIGONOMETRIC FUNCTIONS FOR EVERY DEGREE FROM 0° to 90°

Angle	sin	cos	tan	cot	sec	csc	
0°	.0000	1.0000	.0000		1.0000		90°
1°	.0175	.9998	.0175	57.2900	1.0002	57.2987	89°
2°	.0349	.9994	.0349	28.6363	1.0006	28.6537	88°
3°	.0523	.9986	.0524	19.0811	1.0014	19.1073	87°
4°	.0698	.9976	.0699	14.3007	1.0024	14.3356	86°
5°	.0872	.9962	.0875	11.4301	1.0038	11.4737	85°
6°	.1045	.9945	.1051	9.5144	1.0055	9.5668	84°
7°	.1219	.9925	.1228	8.1443	1.0075	8.2055	83°
8°	.1392	.9903	.1405	7.1154	1.0098	7.1853	82°
9°	.1564	.9877	.1584	6.3138	1.0125	6.3925	81°
10°	.1736	.9848	.1763	5.6713	1.0154	5.7588	80°
11°	.1908	.9816	.1944	5.1446	1.0187	5.2408	79°
12°	.2079	.9781	.2126	4.7046	1.0223	4.8097	78°
13°	.2250	.9744	.2309	4.3315	1.0263	4.4454	77°
14°	.2419	.9703	.2493	4.0108	1.0306	4.1336	76°
15°	.2588	.9659	.2679	3.7321	1.0353	3.8637	75°
16°	.2756	.9613	.2867	3.4874	1.0403	3.6280	74°
17°	.2924	.9563	.3057	3.2709	1.0457	3.4203	73°
18°	.3090	.9511	.3249	3.0777	1.0515	3.2361	72°
19°	.3256	.9455	.3443	2.9042	1.0576	3.0716	71°
20°	.3420	.9397	.3640	2.7475	1.0642	2.9238	70°
21°	.3584	.9336	.3839	2.6051	1.0711	2.7904	69°
22°	.3746	.9272	.4040	2.4751	1.0785	2.6695	68°
23°	.3907	.9205	.4245	2.3559	1.0864	2.5593	67°
24°	.4067	.9135	.4452	2.2460	1.0946	2.4586	66°
25°	.4226	.9063	.4663	2.1445	1.1034	2.3662	65°
26°	.4384	.8988	.4877	2.0503	1.1126	2.2812	64°
27°	.4540	.8910	.5095	1.9626	1.1223	2.2027	63°
28°	.4695	.8829	.5317	1.8807	1.1326	2.1301	62°
29°	.4848	.8746	.5543	1.8040	1.1434	2.0627	61°
30°	.5000	.8660	.5774	1.7321	1.1547	2.0000	60°
31°	.5150	.8572	.6009	1.6643	1.1666	1.9416	59°
32°	.5299	.8480	.6249	1.6003	1.1792	1.8871	58°
33°	.5446	.8387	.6494	1.5399	1.1924	1.8361	57°
34°	.5592	.8290	.6745	1.4826	1.2062	1.7883	56°
35°	.5736	.8192	.7002	1.4281	1.2208	1.7434	55°
36°	.5878	.8090	.7265	1.3764	1.2361	1.7013	54°
37°	.6018	.7986	.7536	1.3270	1.2521	1.6616	53°
38°	.6157	.7880	.7813	1.2799	1.2690	1.6243	52°
39°	.6293	.7771	.8098	1.2349	1.2868	1.5890	51°
40°	.6428	.7660	.8391	1.1918	1.3054	1.5557	50°
41°	.6561	.7547	.8693	1.1504	1.3250	1.5243	49°
42°	.6691	.7431	.9004	1.1106	1.3456	1.4945	48°
43°	.6820	.7314	.9325	1.0724	1.3673	1.4663	47°
44°	.6947	.7193	.9657	1.0355	1.3902	1.4396	46°
45°	.7071	.7071	1.0000	1.0000	1.4142	1.4142	45°
	cos	sin	cot	tan	csc	sec	Angle