APPENDIX A

REFERENCE DATA

A.1 INTRODUCTION

This appendix contains reference material required in connection with the engineering of Microwave and Tropospheric Communication Systems.

A.2 STATISTICS

Appropriate definitions and formulas used in determining Binomial, Normal, and Poisson distributions are given in figure A-1.

A.3 REFERENCE CURVES AND NOMOGRAPHS

Reference curves and nomographs most commonly used in the engineering of Microwave and Tropospheric Systems are given in figures A-2 through A-18.

A.4 EQUATIONS

A compilation of common equations are shown in figure A-19.

A.5 CONVERSION TABLES

Conversion tables of various frequency, wavelength, and metric units are shown in figure A-20.

STATISTICS

A. DEFINITIONS

Arithmetic Mean —

The arithmetic mean of a set of numbers is defined as follows:

$$\overline{X} = \frac{X_1 + X_2 + X_3 \ldots + X_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Weighted Arithmetic Mean —

$$\overline{X} = \frac{W_1X_1 + W_2X_2 + W_3X_3 \ldots W_nX_n}{W_1 + W_2 \ldots W_n}$$

$$\overline{X} = \sum_{i=1}^{n} W_iX_i \Big/ \sum_{i=1}^{n} W_i$$

Standard Deviation —

The standard deviation of a set of numbers is defined as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n}}$$

Variance $= \sigma^2$

Geometric Mean —

The geometric mean of a set of N numbers, $X_1X_2X_3$. X_n is the *nth* root of the product.

$$G = \sqrt[N]{X_1X_2X_3 \ldots X_n}$$

Harmonic Mean —

The harmonic mean of a set of numbers X_1, X_2, $X_3, \ldots X_n$ is the reciprocal of the arithmetic mean of the reciprocal of the numbers:

$$H = \frac{1}{\frac{1}{N}\sum_{j=1}^{N} \frac{1}{X_j}}$$

Root Mean Square —

The root mean square (RMS) of a set of numbers $X_1, X_2 \ldots X$ is defined as follows:

$$RMS = \sqrt{\sum_{j=1}^{N} \frac{X_j^2}{N}}$$

Median —

The median is the middle value, or the arithmetic mean of the two middle values.

Example 1:

3, 4, 4, 5, 6, 8, 8, 8, 10 median = 6

Example 2:

5, 5, 7, 9, 11, 12, 15, 18 median $= \frac{9 + 11}{2} = 10$

Mode —

The mode is the number which occurs with the greatest frequency and may not exist or there may be more than one value:

Example 1:

2, 2, 5, 7, 9, 9, 9, 10, 10, 11, 12, 18; mode = 9

Example 2:

3, 5, 8, 10, 12, 15, 16; no mode

B. BINOMIAL DISTRIBUTION

If p is the probability that an event will happen in any single trial (called the probability of a *success*)

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Figure A-1. Statistics (Sheet 1 of 3)

and $q = 1 - p$ is the probability that it will fail to happen in any single trial (called the probability of a *failure*) then the probability that the event will happen exactly X times in N trials (*i.e.,* X successes and $N - X$ failures will occur) is given by

$$p(X) = {}_NC_X p^X q^{N-X} = \frac{N!}{X!\,(N-X)!} p^X q^{N-X}$$

where $X = 0, 1, 2, \ldots, N$ and $N! = N(N-1)(N-2)\ldots 1.$ $0! = 1$ by definition.

Example 1: The probability of getting exactly 2 heads in 6 tosses of a fair coin is

$${}_6C_2(\tfrac{1}{2})^2(\tfrac{1}{2})^{6-2} = \frac{6!}{2!\,4!}(\tfrac{1}{2})^6 = {}^{15}\!/_{64}$$

with $N = 6$, $X = 2$, and $p = q = \frac{1}{2}$.

Example 2: The probability of getting at least 4 heads in 6 tosses of a fair coin is

$${}_6C_4(\tfrac{1}{2})^4(\tfrac{1}{2})^{6-4} + {}_6C_5(\tfrac{1}{2})^5(\tfrac{1}{2})^{6-5} + {}_6C_6(\tfrac{1}{2})^6 = {}^{15}\!/_{64} + {}^{6}\!/_{64} + {}^{1}\!/_{64} = {}^{11}\!/_{32}$$

Some properties of the binomial distribution are listed as follows:

Mean	$\mu = Np$
Variance	$\sigma^2 = Npq$
Standard deviation	$\sigma = \sqrt{Npq}$
Moment coefficient of skewness	$\alpha_3 = \frac{q-p}{\sqrt{Npq}}$
Moment coefficient of kurtosis	$\alpha_4 = 3 + \frac{1-6pq}{Npq}$

C. THE NORMAL DISTRIBUTION

One of the most important examples of a probability distribution is the ***normal distribution, normal curve*** or ***Gaussian distribution*** defined by the equation

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$$Y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\tfrac{1}{2}(X-\mu)^2/\sigma^2\right]$$

where μ = mean, σ = standard deviation, $\pi = 3.14159\ldots$, $e = 2.71828\ldots$

The total area bounded by the curve and the X axis is one; hence the area under the curve between two ordinates $X = a$ and $X = b$, where $a < b$, represents the probability that X lies between a and b, denoted by $Pr\{a < X < b\}$.

When the variable X is expressed in terms of standard units, $z = (X-\mu)/\sigma$, the so-called ***standard form*** is expressed as

$$Y = \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}z^2\right)$$

In such case we say that z is ***normally distributed with mean zero and variance one.***

A graph of this standardized normal curve as shown in Figure 114 has indicated the areas included between $z = -1$ and $+1$, $z = -2$ and $+2$, $z = +3$ and -3 are equal respectively to 68.27%, 95.45% and 99.73% of the total area.

Example of Normal Distribution

The mean weight of 500 students is 151 lbs and $\sigma = 15$; assume normal distribution, determine how many students weigh (a) between 120 and 155 lbs and (b) more than 185 lbs.

a.

$$Z = \frac{(119.5 - 151)}{15} = -2.10$$

$$Z = \frac{(155.5 - 151)}{15} = .30$$

Area from -2.10 to $.30 = 0.600$

The number of students = 500 (0.600) = 300

b. Students weighing more than 185

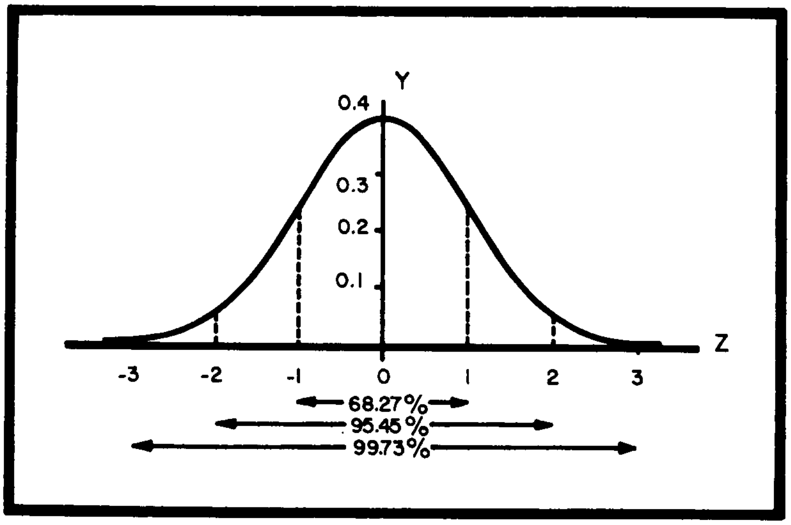
$$Z = \frac{185.5 - 151}{15} = 2.30$$

Area between 0, and 2.30 = 0.4983

= 0.5 − 0.4893 = 0.0107

The number of students = 500 (0.0107) = 5

Figure A-1. Statistics (Sheet 2 of 3)



MEAN	μ
VARIANCE	σ^2
STANDARD DEVIATION	σ
MOMENT COEFFICIENT OF SKEWNESS	$\alpha_3 = 0$
MOMENT COEFFICIENT OF KURTOSIS	$\alpha_4 = 3$
MEAN DEVIATION	$\sigma \sqrt{2/\pi} = 0.7979$

Some Properties of the Normal Distribution

D. RELATION BETWEEN BINOMIAL AND NORMAL DISTRIBUTIONS

If N is large and if neither p nor q is too close to zero, the binomial distribution can be closely approximated by a normal distribution with standardized variable given by

$$z = \frac{X - Np}{\sqrt{Npq}}.$$

E. THE POISSON DISTRIBUTION

The discrete probability distribution

$$p(X) = \frac{\lambda^X e^{-\lambda}}{X!} \qquad (X = 0, 1, 2, \ldots)$$

where e = 2.71828 . . . and λ is a given constant, is called the *Poisson distribution.*

Some properties of the Poisson distribution are listed in the following table.

Mean	$\mu = \lambda$
Variance	$\sigma^2 = \lambda$
Standard deviation	$\sigma = \sqrt{\lambda}$
Moment coefficient of skewness	$\alpha_3 = 1/\sqrt{\lambda}$
Moment coefficient of kurtosis	$\alpha_4 = 3 + 1\,\lambda$

F. RELATION BETWEEN BINOMIAL AND POISSON DISTRIBUTIONS

In the binomial distribution, if N is large while the probability p of occurrence of an event is close to zero so that q = (1 − p) is close to 1, the event is called a *rare event.* In practice we shall consider an event as rare if the number of trials is at least 50 ($N \geq 50$) while Np is less than 5. In such cases the binomial distribution is very closely approximated by the Poisson distribution with $\lambda = Np$.

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Figure A-1. Statistics (Sheet 3 of 3)

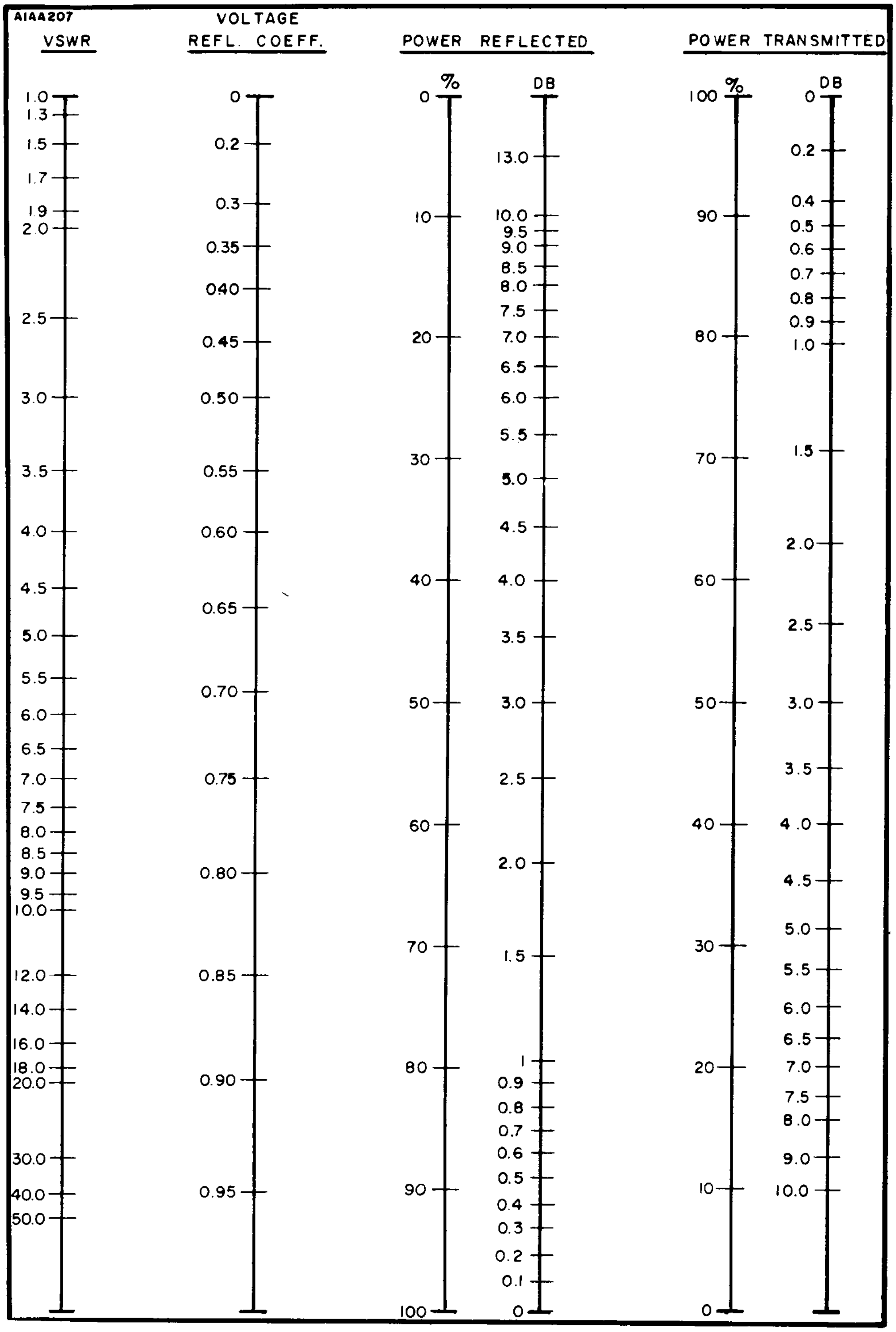

Figure A-2. VSWR Nomograph #1

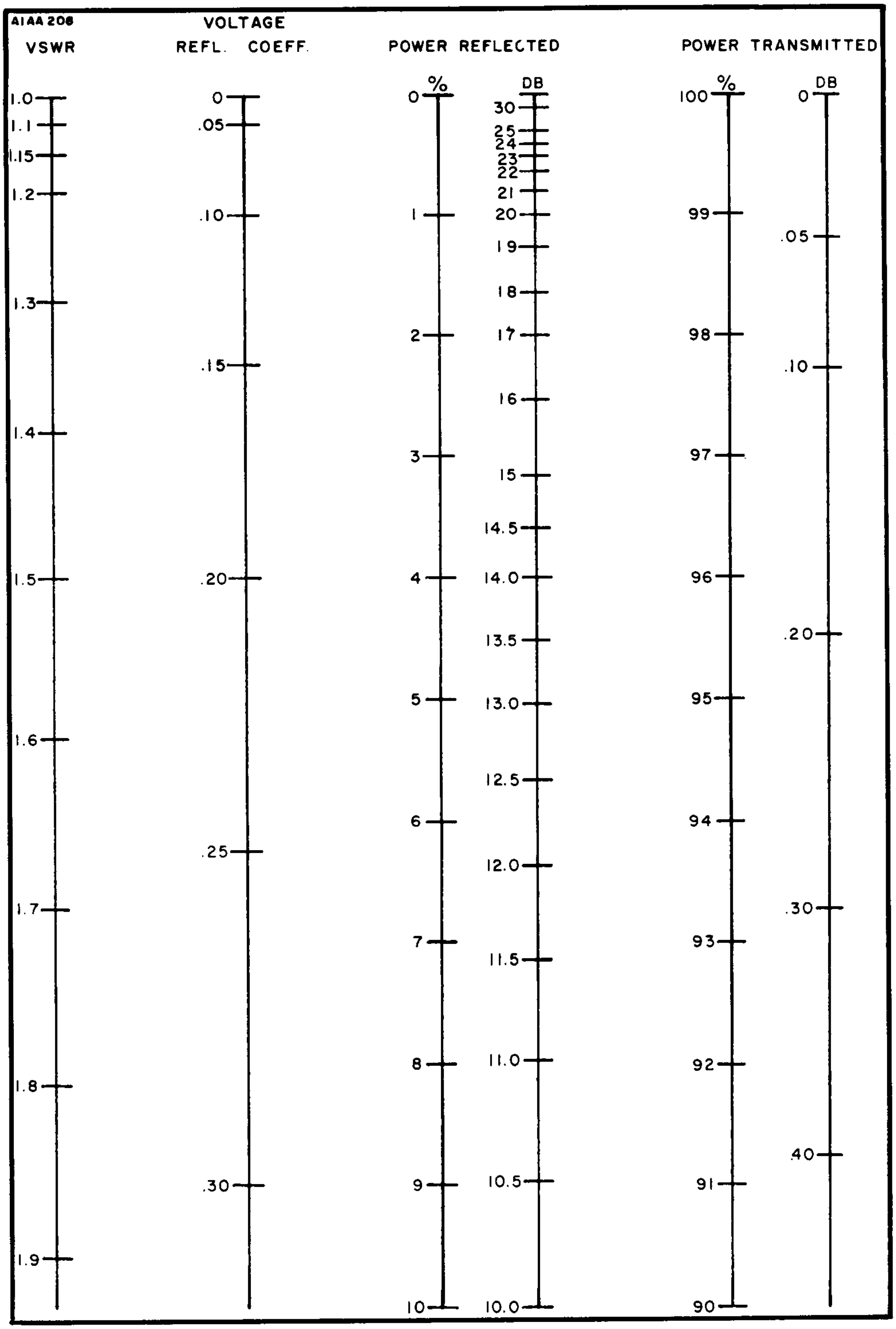

Figure A-3. VSWR Nomograph #2

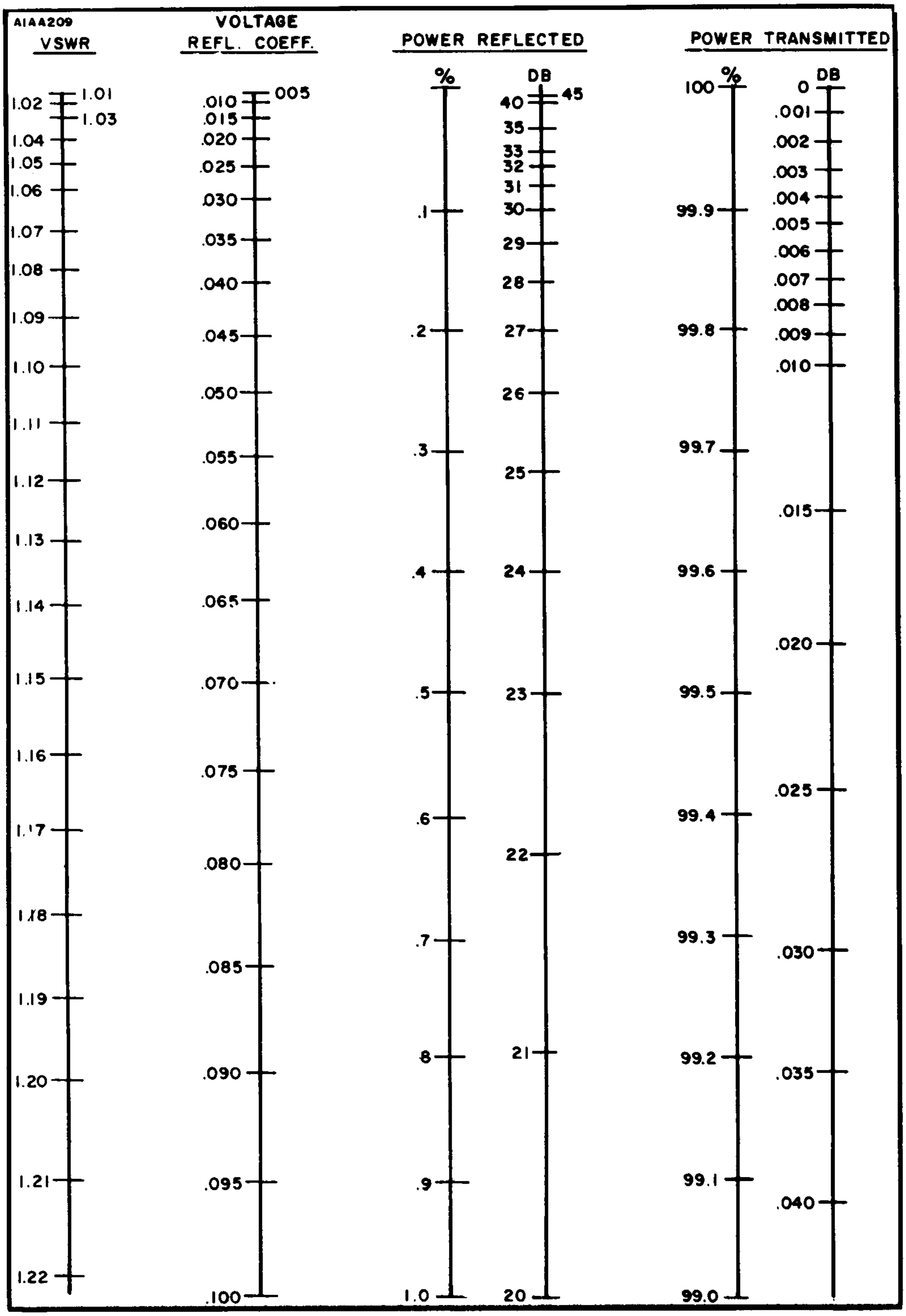

Figure A-4. VSWR Nomograph #3

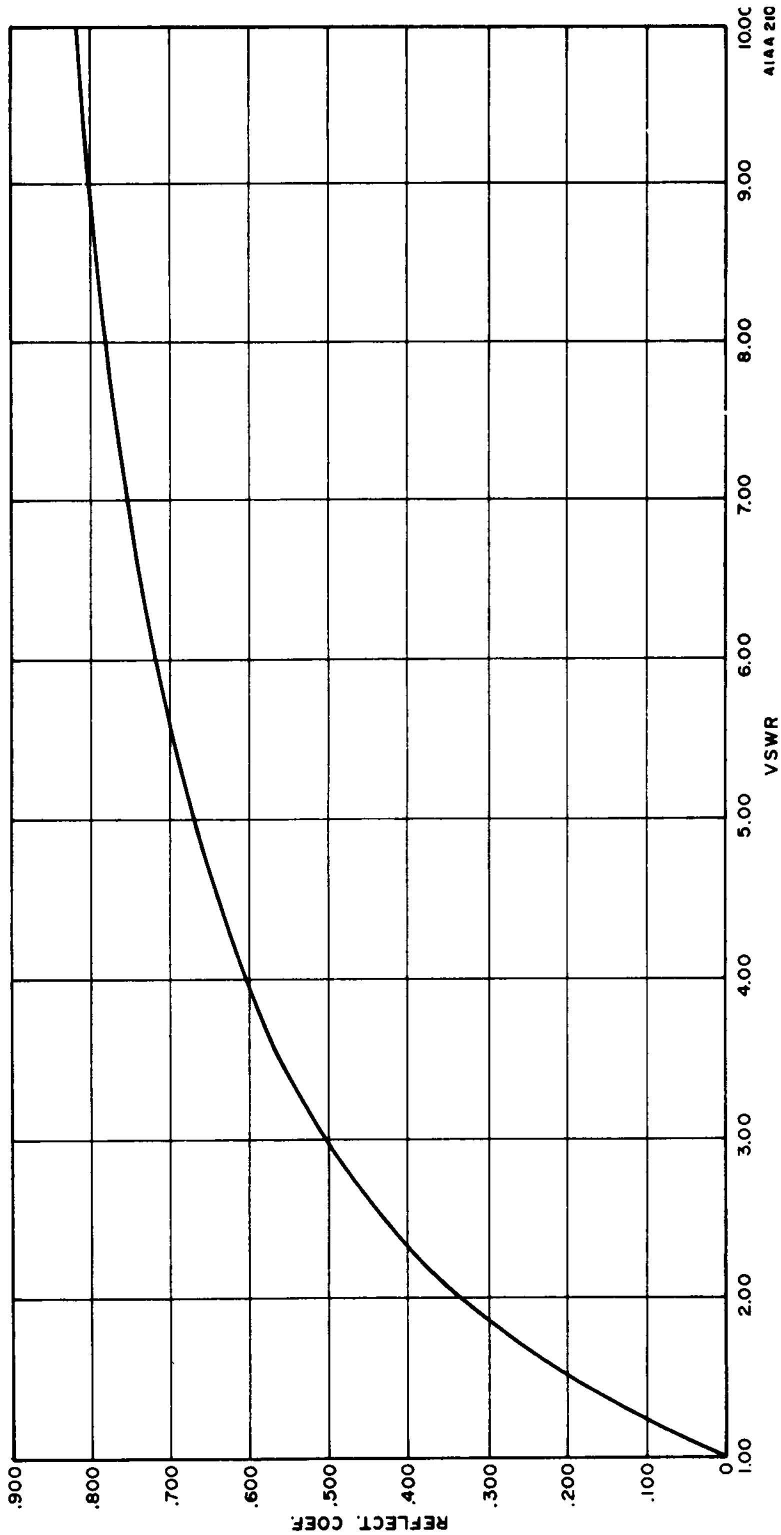

Figure A-5. VSWR Versus Reflection Coefficient

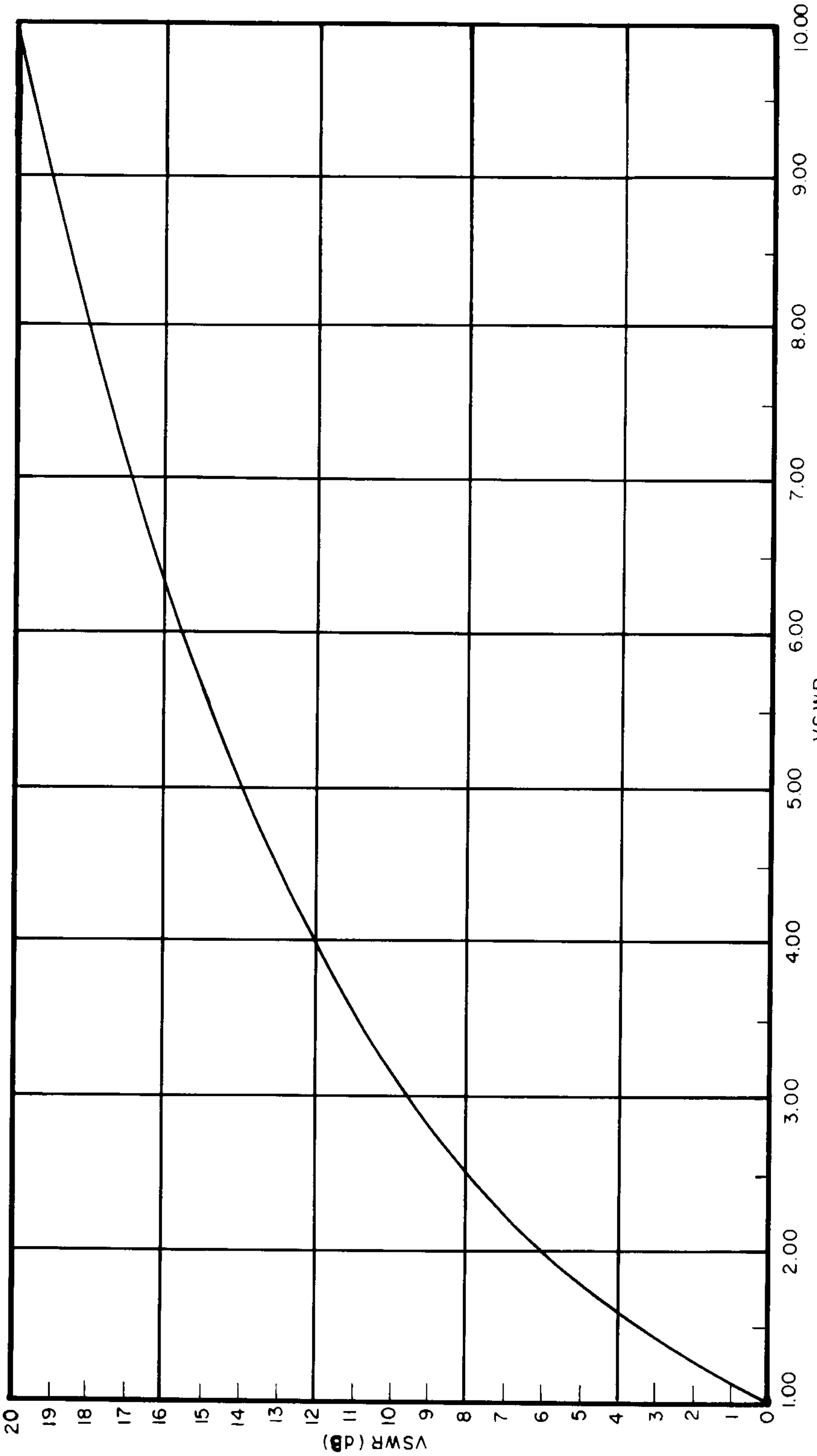

Figure A-6. VSWR Versus VSWR (dB)

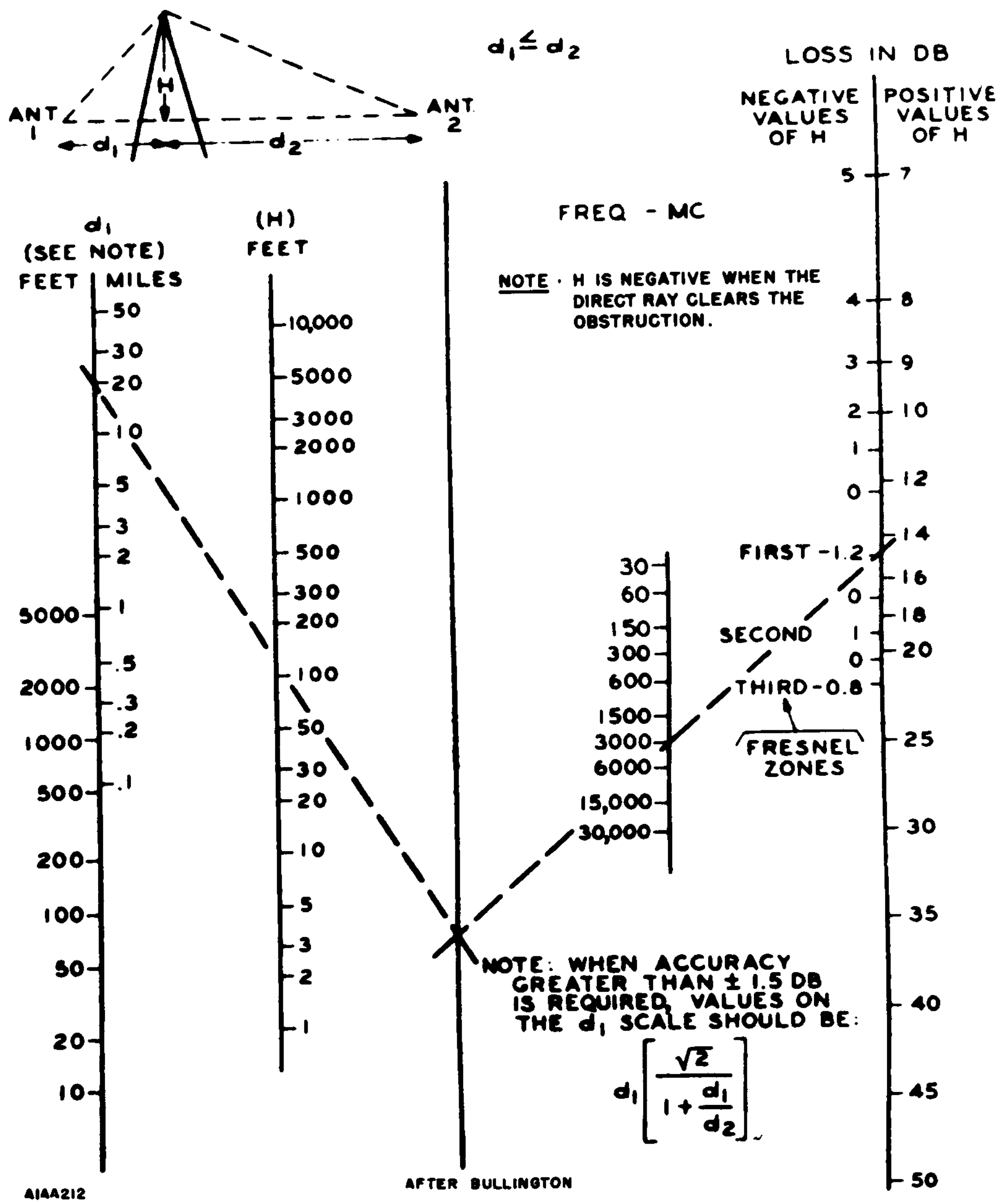

Figure A-7. Knife-Edge Diffraction Relation to Free Space

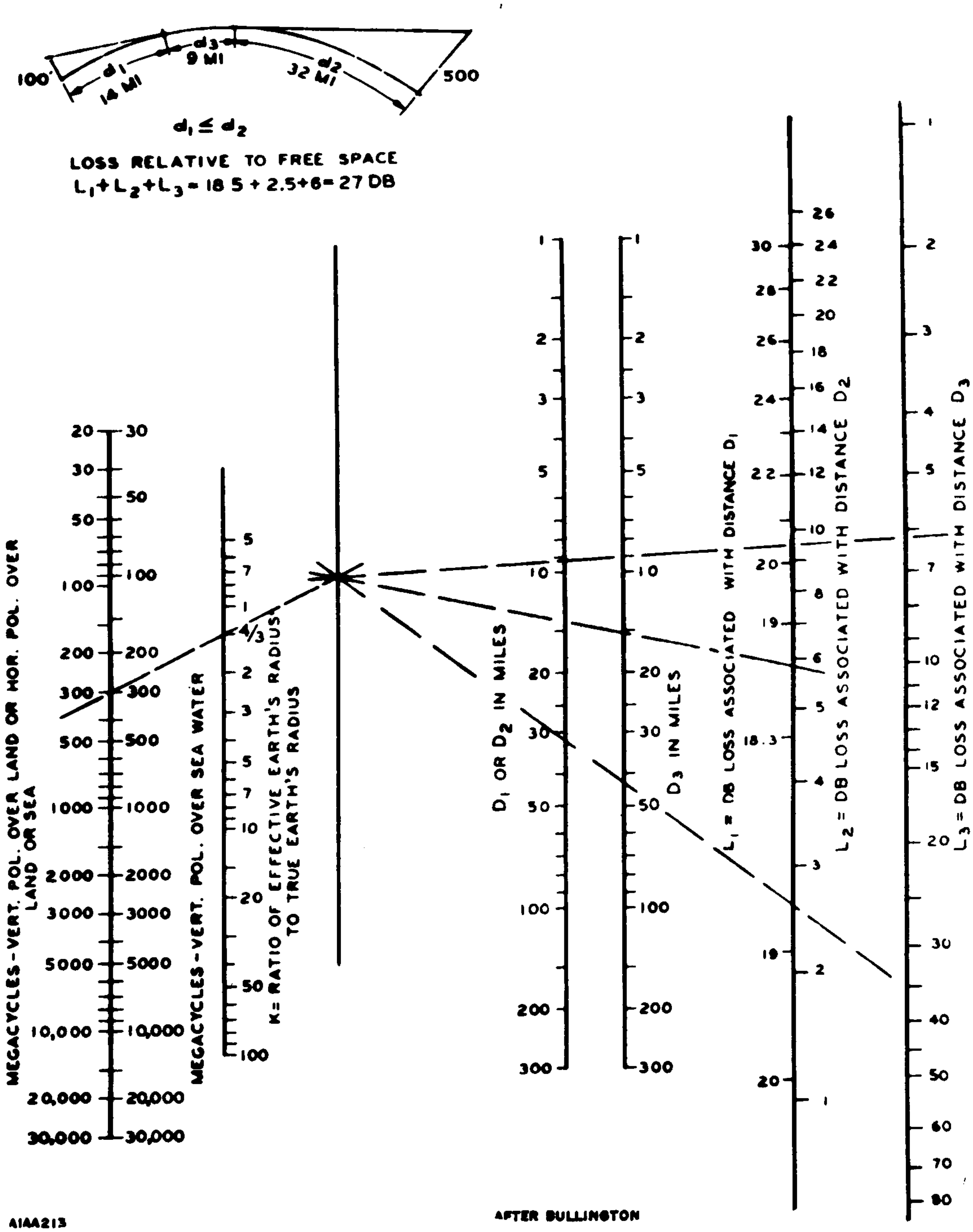

Figure A-8. Diffraction Loss Relative to Free Space Transmission at all Locations Beyond Line-of-Sight Over a Smooth Sphere

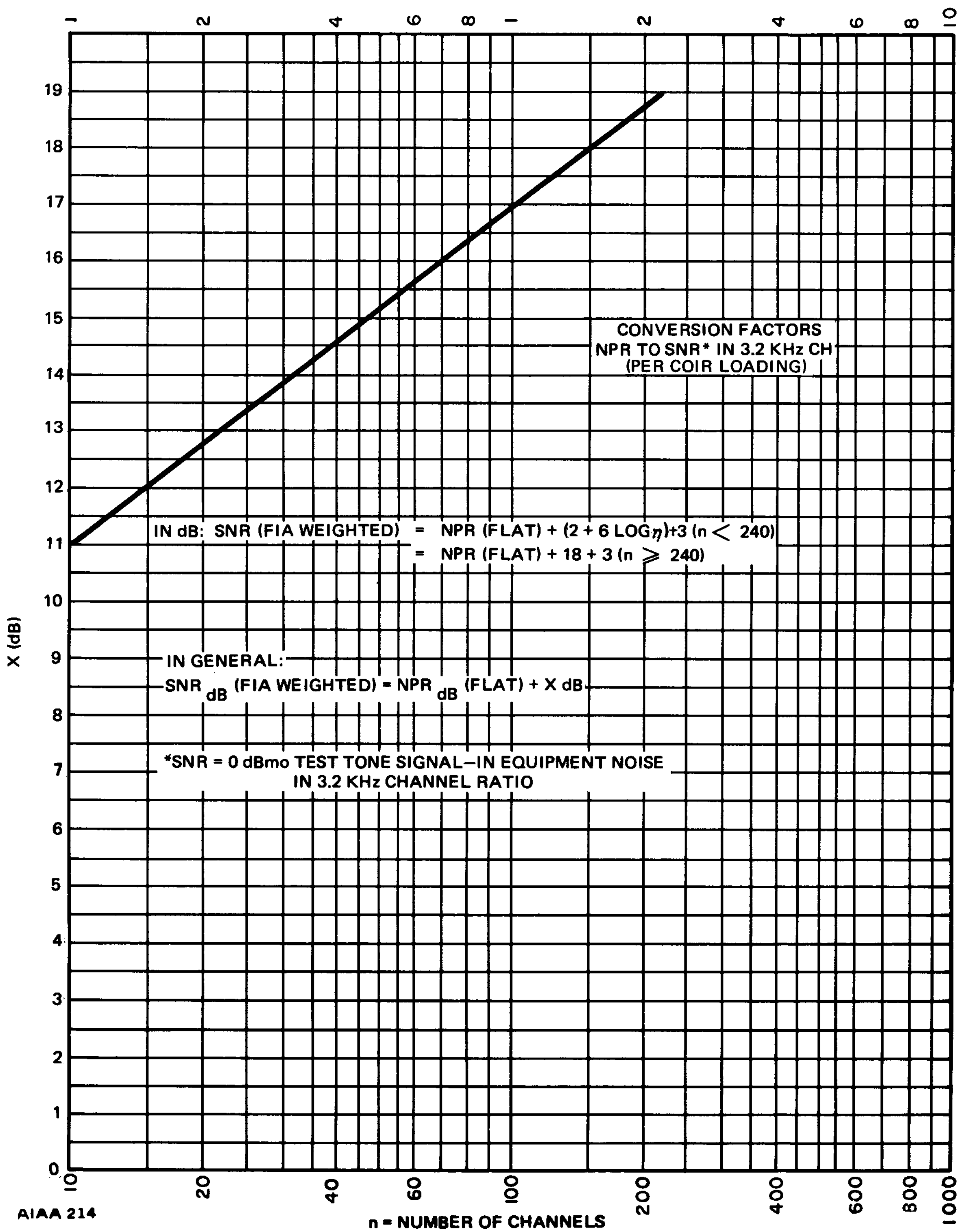

Figure A-9. Conversion Factors, NPR to SNR

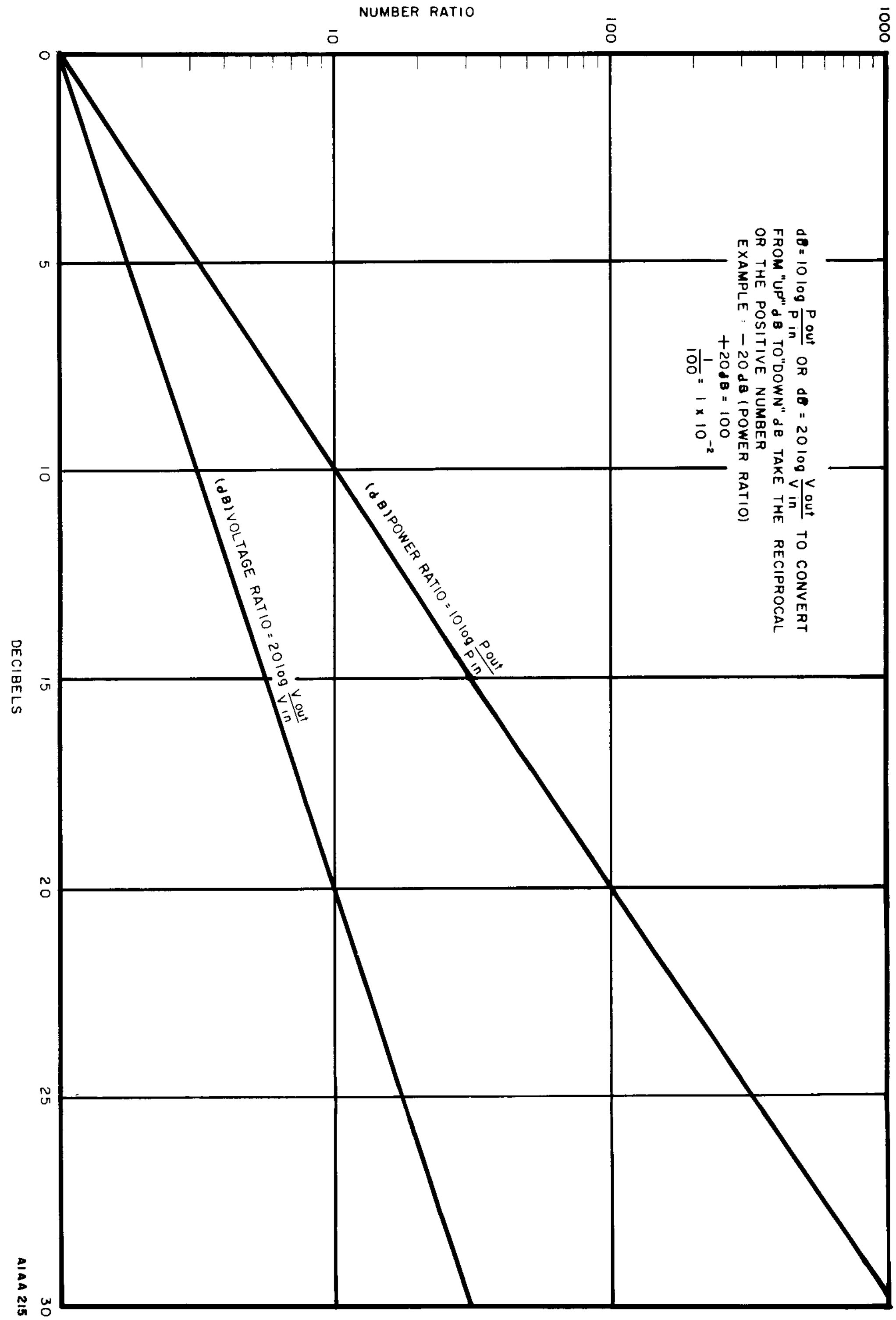

Figure A-10. Power Ratio and Voltage Ratio in Natural Numbers and Logarithms

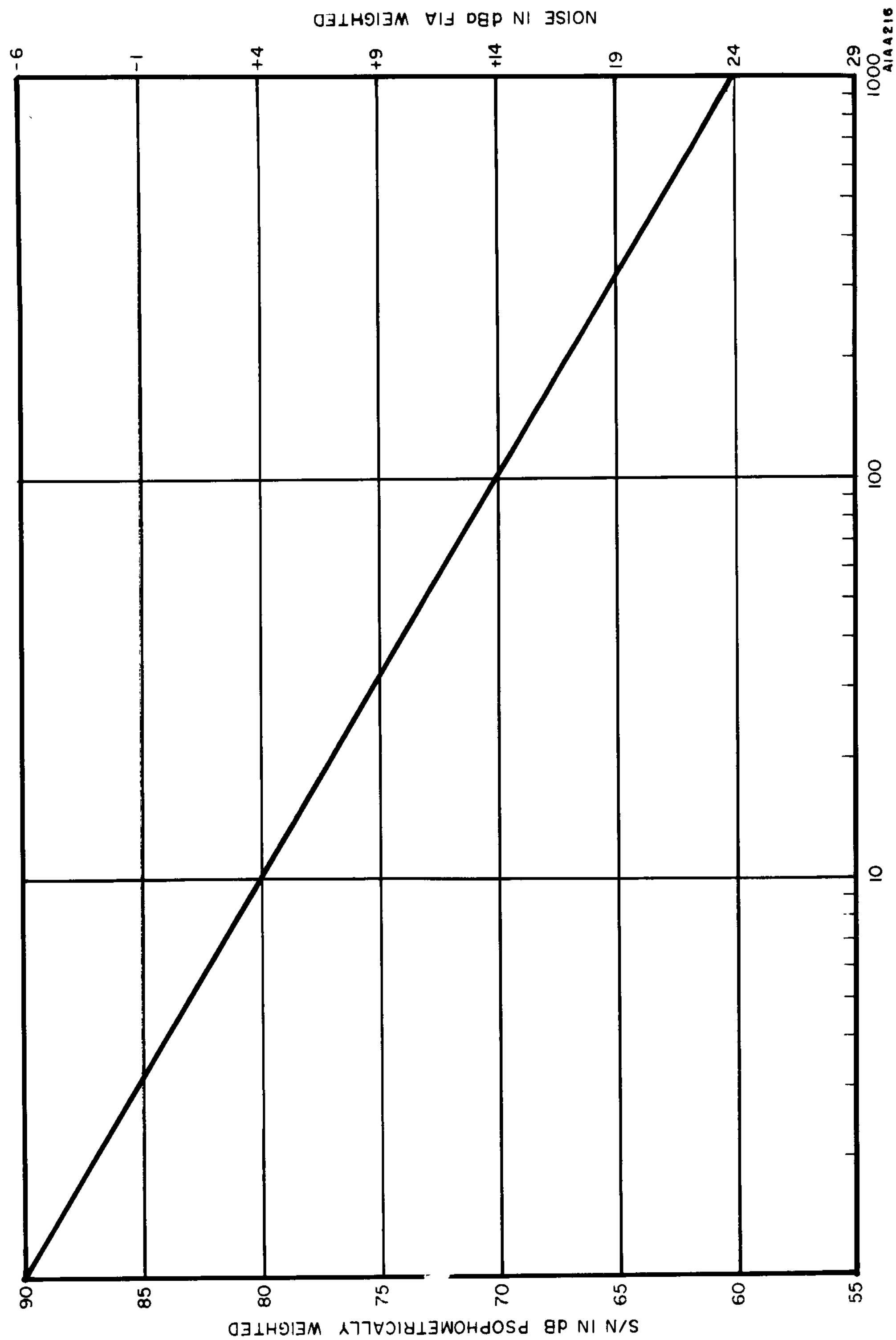

Figure A-11. Noise in Picowatts Psophometrically Weighted #1

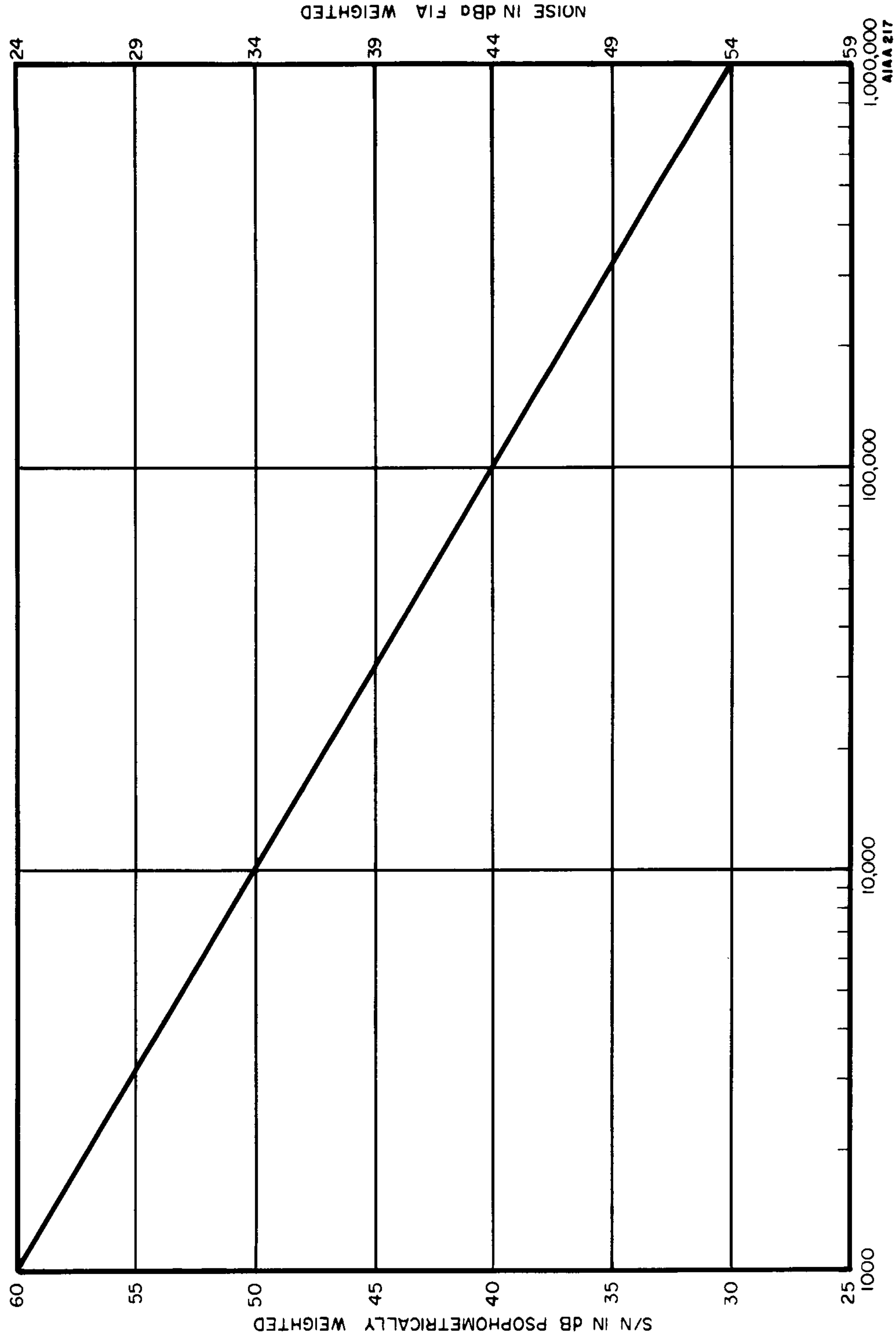

Figure A-12. Noise in Picowatts Psophometrically Weighted #2

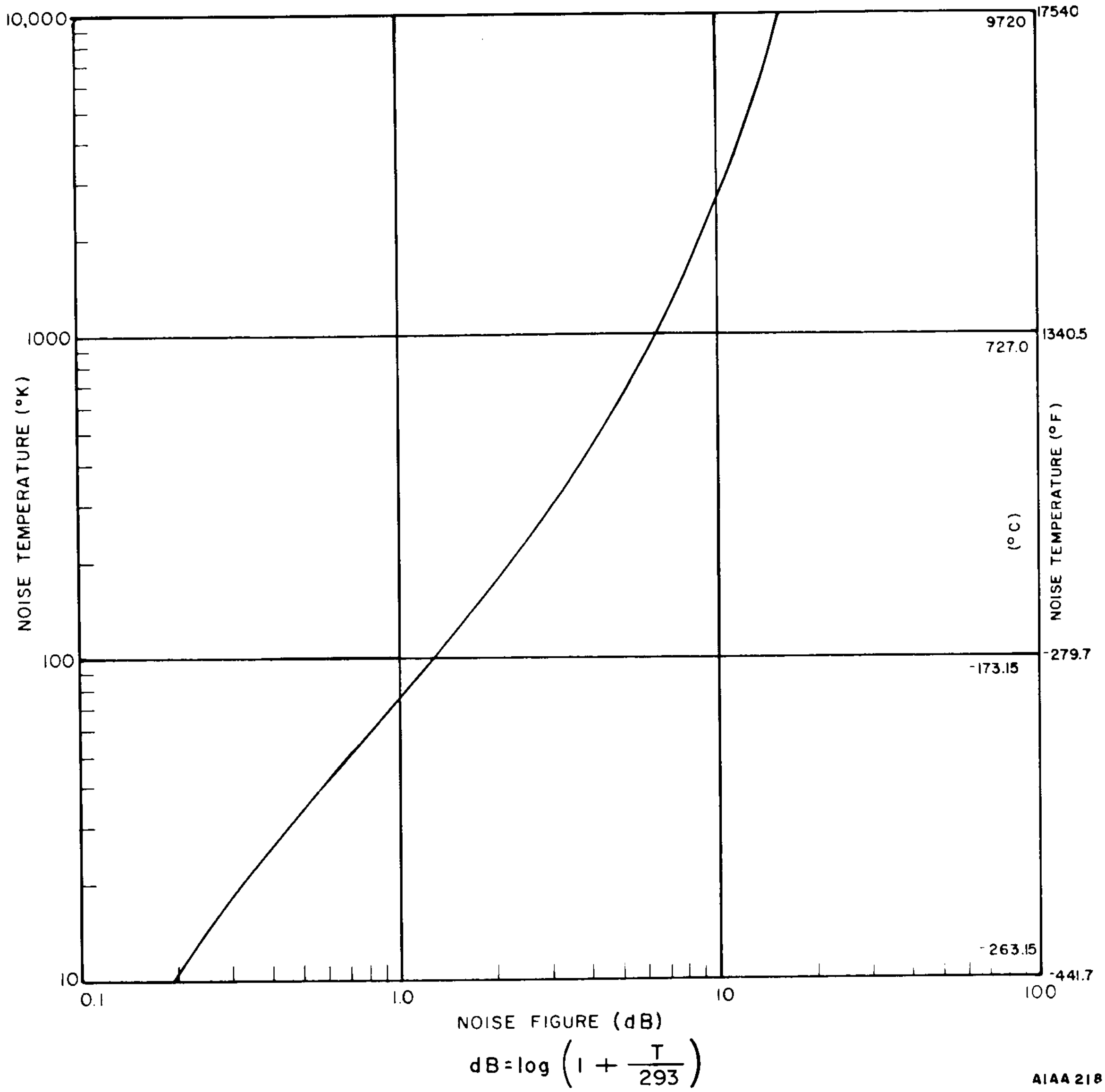

Figure A-13. Noise Figure (293°K) Versus Noise Temperature (°K)

$$dB = 10 \log \left(1 + \frac{T}{293} \right)$$

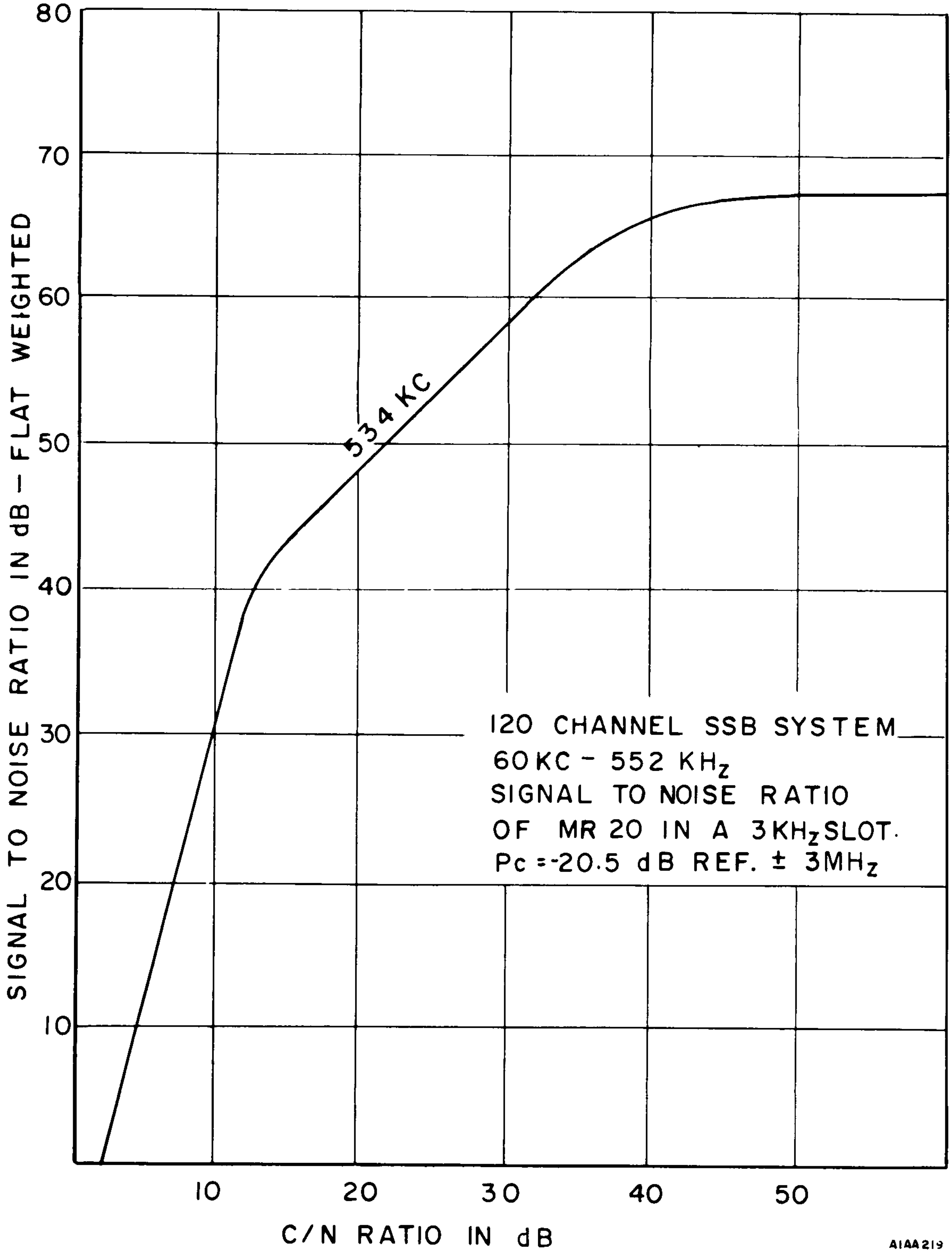

Figure A-14. FM Receiver Characteristic Curves

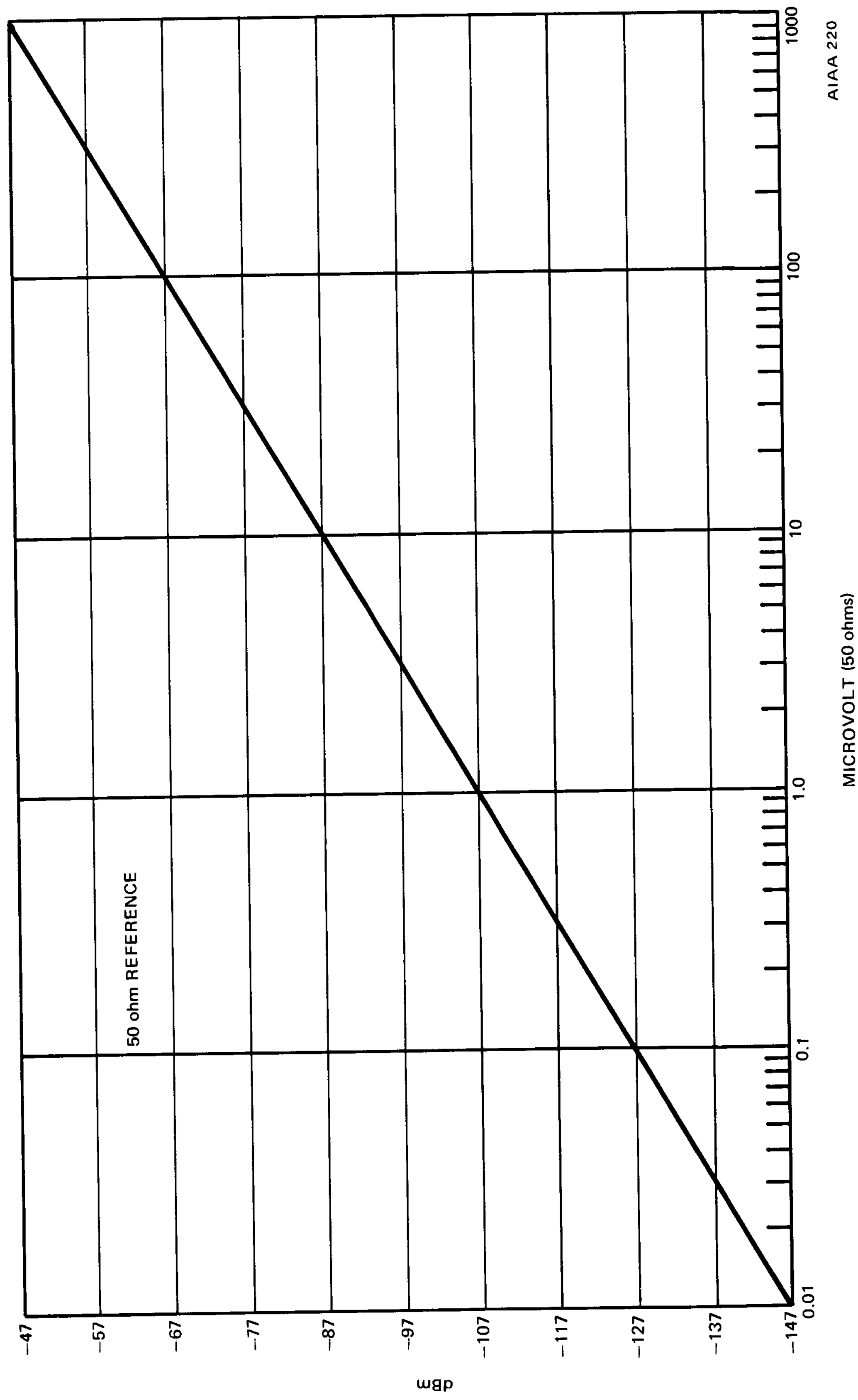

Figure A-15. Microvolt/dbm Conversion

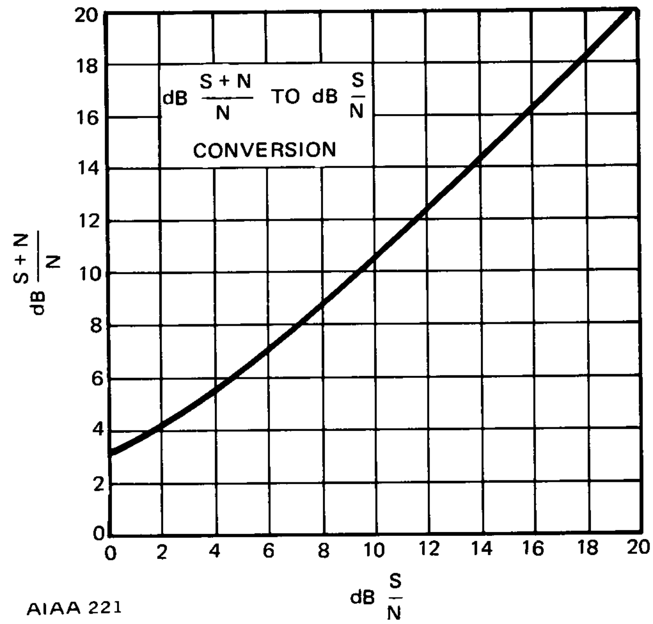

Figure A-16. Conversion of S + N/N (dB) to S/N (dB)

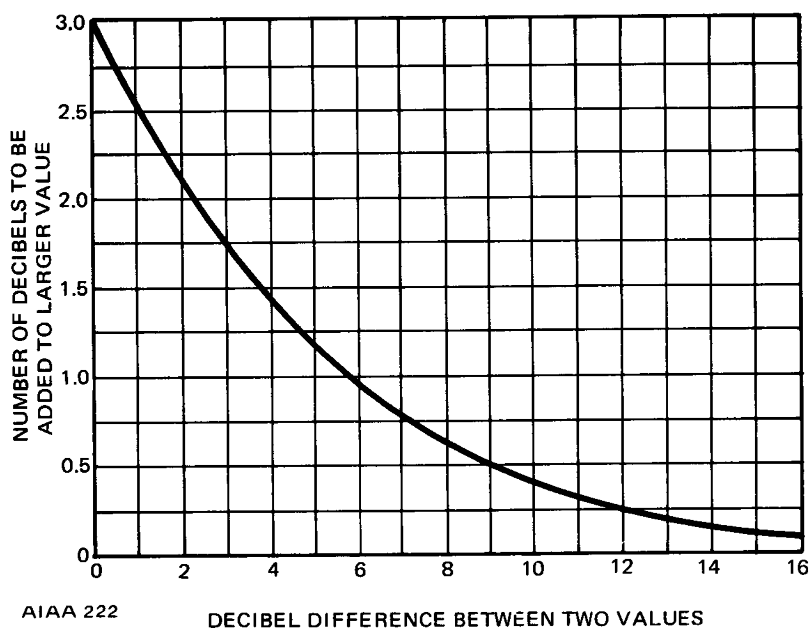

Figure A-17. Addition of Noise

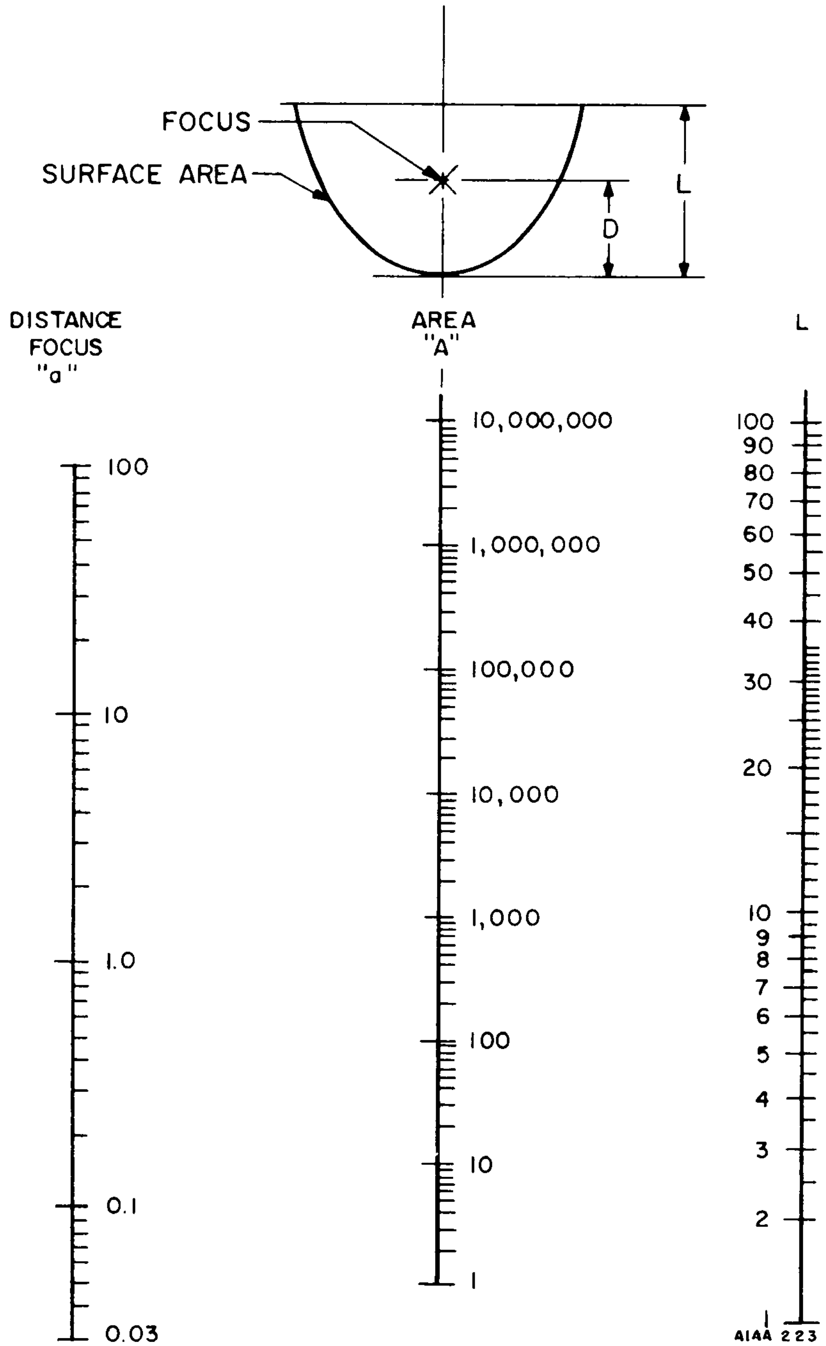

Figure A-18. Nomograph for Determining Surface Areas of Paraboloid Devices

Loading CCIR Standard

L (mean) $= -1 + 4 \log N$ $\quad N < 240$

L (mean) $= -15 + 10 \log N$ $\quad N \geq 240$

Loading Military Standard

L (mean) $= -5 + 10 \log N$

S/N (thermal) $= C/N + 10 \log \frac{B_{if}}{2b} + 20 \log \frac{\Delta F}{Fm} - L\text{ (peak)} + P + W + C$

S/N (intermodulation) $= NPR + 10 \log \times \frac{B_b}{b}$ − NLR (Noise Loading Ratio)

dBa = dBm + 85 F1A (1000 cycle reference)

dBa = dBm + 82 flat weighted (one voice channel)

dBa = dBm + 90 144 weighted

dBa = dBm + 90 C message weighted (1000 cycle reference)

dBa = − S/N + 82 F1A weighted, 144 weighted (1000 cycle reference)

dBrm (C message) = − S/N + 88.5 (1000 cycle reference)

FM Threshold $= 10 \text{ Log } KTB + N_F + 10$

Noise Threshold $= 10 \text{ Log } KTB + N_F$

System Noise Figure $= N_{f1} + \frac{N_{f2} - 1}{G_1} + \frac{N_{f3} - 1}{G_1 G_2} \cdots \frac{N_{fn} - 1}{G_1 G_2 \ldots G_{n-1}}$

$°R = °F + 459.67$

$°F = 9/5\ °C + 32$

$K = 5/9\ R$

$\log MN = \log M + \log N$

$\log M/N = \log M - \log N$

$\log M^P = P \log M$

$$S/N(dB) = \text{Log}\left[\text{Log}^{-1} \frac{S + N(dB)}{N} - 1\right]$$

$$\frac{S + N}{N}(dB) = \text{Log}\left[\text{Log}^{-1} \frac{S}{N}(dB) + 1\right]$$

Equivalent Noise Temperature = Antilog insertion loss × 290°K

$B_{IF} = 3.4 (F_m + \Delta F)$; empirical approx.

$B_{IF} = F_m \times S_1$

$$D = \frac{\Delta F}{F_m}$$

$$S/N\ (P) = 10 \text{ Log } \frac{1 \times 10^{-3}}{Np}$$

(when the signal power is 1 mw)

Allowable noise (pwp) $= \frac{l}{6000} \times 20{,}000 \times 10^{-12}$

$$\text{Log}^{-1}\left[\frac{dBa + 6}{10}\right] = pwp$$

$pw = 1 \times 10^{-12}$ watts

$$L\text{ (peak)} = 20 \log \frac{\Delta F\text{ (peak)}}{\Delta F_b}$$

$$L\text{ (mean)} = 20 \text{ Log } \frac{\Delta F\text{ (rms)}}{\Delta F_b}$$

$\frac{\Delta F}{F_m} = 2.404$ (Carrier goes through first null, all transmitted power in sidebands)

temperature

$°C = 5/9\ (°F - 32)$

$°K = °C + 273.15$

Conversion from dBm to Volts

$$E = \sqrt{\text{Log}^{-1} \frac{dBm}{10} \times R \times 10^{-3}}$$

Conversion from Volts to dBm

$$dBm = 10 \text{ Log } \frac{E^2}{R \times 10^{-3}}$$

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Figure A-19. Common Equations

A. MICROWAVE FREQUENCIES AND CORRESPONDING WAVEGUIDE SIZES

BAND	FITS WAVEGUIDE SIZE (inches)	RANGE FREQUENCY (KMC)
"S"	3 × 1½	2.6 to 3.95
"G"	2 × 1	3.95 to 5.85
"J"	1½ × ¾	5.3 to 8.2
"H"	1¼ × ⅝	7.05 to 10.0
"X"	1 × ½	8.2 to 12.4
"P"	0.702 × 0.391	12.4 to 18.0
"K"	0.500 × 0.250	18.0 to 26.5
"R"	0.360 × 0.220	26.5 to 40

B. CONVERSION TABLE WAVELENGTH FREQUENCY BANDS

WAVELENGTH BAND (meters)	FREQUENCY BAND (kilocycles)	APPROXIMATE NUMBER OF KILOCYCLES PER METER CHANGE IN WAVELENGTH	APPROXIMATE NUMBER OF METERS PER KILOCYCLE CHANGE IN FREQUENCY	OFFICIAL FCC BAND ABBREVIATION (frequency)
VERY LONG WAVES (Infinity to 10,000)	0 to 30	Below 0.01	Over 333	VERY LOW (VLF)
LONG WAVES (10,000 to 1000)	30 to 300	0.05	20	LOW (LF)
MEDIUM WAVES (1000 to 100)	300 to 3000	5	0.2	MEDIUM (MF)
SHORT WAVES (100 to 10)	3000 to 30,000	500	0.002	HIGH (HF)
VERY SHORT WAVES (10 to 1)	30,000 to 300,000	50,000	0.00002	VERY HIGH (VHF)
ULTRA SHORT WAVES (1 to 0.1) (microwaves)	300,000 to 3,000,000	5,000,000	0.0000002	ULTRA HIGH (UHF)
SUPER SHORT WAVES (0.1 to 0.01) (microwaves)	3,000,000 to 30,000,000	500,000,000	0.000000002	SUPER HIGH (SHF)

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Figure A-20. Conversion Tables (Sheet 1 of 2)

C. CONVERSION TABLE OF METRIC UNITS

METER	DECIMETER	CENTIMETER	MILLIMETER	MICRON	MILLI-MICRON	ANGSTROM UNIT
1 METER	10	100	1,000	1,000,000	1,000,000,000	10,000,000,000
0.1	1 DECIMETER	10	100	100,000	100,000,000	1,000,000,000
0.01	0.1	1 CENTIMETER	10	10,000	10,000,000	100,000,000
0.001	0.01	0.1	1 MILLIMETER	1,000	1,000,000	10,000,000
0.000001	0.00001	0.0001	0.001	1 MICRON	1,000	10,000
0.000000001	0.00000001	0.0000001	0.000001	0.001	1 MILLI-MICRON	10
0.0000000001	0.000000001	0.00000001	0.0000001	0.0001	0.1	1 ANGSTROM UNIT

D. WAVELENGTHS OF VARIOUS RADIATION REGIONS

REGION	WAVELENGTH LIMITS (centimeters) Maximum	WAVELENGTH LIMITS (centimeters) Minimum	FREQUENCY LIMITS (kilo-megacycles) Minimum	FREQUENCY LIMITS (kilo-megacycles) Maximum	REMARKS
RADIO	3,000,000	0.1	0.0001	300	VLF/LF/MF/ HF/VHF/UHF/SHF/Exp.
INFRA-RED	0.1	0.00008	300	375,000	Heat & Black Light
LIGHT (visible)	0.00008	0.000038	375,000	790,000	Starts with Red, progresses through Orange, Yellow, Green, Blue & Violet
ULTRA-VIOLET	0.000038	0.0000012	790,000	22,500,000	Chemical & invisible
X-RAYS	0.0000012	0.0000000006	22,500,000	45,000,000,000	——
GAMMA-RAYS	0.000000014	0.0000000001	45,000,000,000	270,000,000,000	Radioactive
COSMIC RAYS	0.0000000001	indefinite	270,000,000,000	indefinite	Little known

E. CONVERSION TABLE OF METRIC UNITS INTO WAVELENGTH UNITS

WAVELENGTH UNIT	EQUIVALENCE IN INCHES	EQUIVALENCE IN KILOCYCLES
1 meter	39.37″	300,000
1 decimeter	3.937″	3,000,000
1 centimeter	0.3937″	30,000,000
1 millimeter	0.03937″	300,000,000
1 micron	0.00003937″	300,000,000,000
1 milli-micron	0.00000003937″	300,000,000,000,000
1 angstrom unit	0.000000003937″	3,000,000,000,000,000

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Figure A-20. Conversion Tables (Sheet 2 of 2)